The (μ^-, e^+) conversion in nuclei mediated by light Majorana neutrinos

Fedor Šimkovic[a,b], Pavol Domin[b], Sergey Kovalenko[c 1], Amand Faessler[a]

[a] Institute für Theoretische Physik der Universität Tübingen, Auf der Morgenstelle 14, D-72076 Tübingen, Germany

[b] Department of Nuclear physics, Comenius University, SK–842 15 Bratislava, Slovakia

[c] Departamento de Física, Universidad Técnica Federico Santa María, Casilla 110-V, Valparaíso, Chile

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Abstract

We study the lepton number violating (μ^-, e^+) conversion in nuclei mediated by the exchange of virtual light Majorana neutrinos. We found that a previously overlooked imaginary part of this amplitude plays an important role. The numerical calculation has been made for the experimentally interesting (μ^-, e^+) conversion in ^{48}Ti using realistic renormalized proton-neutron QRPA wave functions. We also discuss the very similar case of the neutrinoless double beta decay of ^{48}Ca. The ratio of (μ^-, e^+) conversion over the total μ^- absorption has been computed taking into account the current constraints from neutrino oscillation phenomenology. We compare our results with the experimental limits as well as with previous theoretical predictions. We have found that the Majorana neutrino mode of (μ^-, e^+) conversion in ^{48}Ti is too small to be measurable in the foreseeable future.

1 Introduction

Lepton number (L) conservation is one of the most obscure sides of the standard model(SM) not supported by an underlying principle and following from an accidental interplay between gauge symmetry and field content. Any deviation from the SM structure may introduce L non-conservation($\not{L}$). Over the years the possibility of lepton number non-conservation has been attracting a great deal of theoretical and experimental efforts since any positive experimental $\not{L}$ signal would request physics beyond the SM. In addition it would also show, that neutrinos are Majorana particles [1].

Recent neutrino oscillation experiments practically established the presence of non-zero neutrino masses, a fact that itself points to physics beyond the SM. However neutrino oscillations are not sensitive to the nature of neutrino masses: they can either be Majorana or Dirac masses leading to the same observables.

The principal question if neutrinos are Majorana or Dirac particles can be answered only by studying a lepton number violating processes since $\Delta L = 2$ is a generic tag of Majorana neutrinos. Various lepton-number violating processes have been discussed in the literature in this respect (for recent review see [2]). They offer the possibility of probing different entries of the Majorana neutrino mass matrix $M^{(\nu)}_{ij}$. Among them there are a few $\not{L}$ nuclear processes having prospects for experimental searches: neutrinoless double beta decay $(0\nu\beta\beta)$, muon

[1] On leave of absence from the Joint Institute for Nuclear Research, Dubna, Russia

to positron (μ^-, e^+) conversion and, probably, muon to antimuon (μ^-, μ^+) conversion [3]. They probe $M^{(\nu)}_{ee}$, $M^{(\nu)}_{\mu e}$ and $M^{(\nu)}_{\mu\mu}$ matrix elements respectively. Currently the most sensitive experiment intended to distinguish the Majorona nature of neutrinos are those searching for neutrinoless $0\nu\beta\beta$-decay [4, 5, 6] The nuclear theory side [7, 8] of this process have been significantly improved in the last decade [8] allowing reliable extraction of fundamental particle physics parameters from experimental data. Muon to positron nuclear conversion (μ^-, e^+) is another $\not{L}$ nuclear process with good experimental prospects.

The important role of muon as a test particle in the search for new physics beyond the standard model has been recognized long time ago. When negative muons penetrate into matter they can be trapped to atomic orbits. Then the bound muon can disappear either decaying into an electron and two neutrinos or being captured by the nucleus, i.e., due to ordinary muon capture. These two processes conserving both total lepton number and lepton flavors have been well studied both theoretically and experimentally. However, there are two other not yet observed channels of muon capture: The muon-electron (μ^-, e^-) and muon-positron (μ^-, e^+) conversions in nuclei [9, 10, 11, 12, 13, 14, 15]:

$$\begin{aligned} (A,Z) + \mu_b^- &\rightarrow e^- + (A,Z)^*, \\ (A,Z) + \mu_b^- &\rightarrow e^+ + (A,Z-2)^*. \end{aligned} \tag{1}$$

Apparently, the (μ^-, e^+) and (μ^-, e^-) conversion processes violate lepton number L and lepton flavor L_f conservation respectively. Additional differences between the (μ^-, e^-) and (μ^-, e^+) lie on the nuclear physics side. The first process can proceed on one nucleon of the participating nucleus while the second process involves two nucleons as dictated by charge conservation [10, 11]. Note also that the (μ^-, e^-) conversion amplitude is quadratic and (μ^-, e^+) amplitude linear in the neutrino mass. Thus the second process looks more sensitive to the light neutrino masses. The present experimental limit on (μ^-, e^+) conversion branching ratio in ^{48}Ti is [16, 17]

$$R_{\mu e^+}(Ti) = \frac{\Gamma(\mu^- +^{48} Ti \rightarrow e^+ +^{40} Ca)}{\Gamma(\mu^- +^{48} Ti \rightarrow \nu_\mu +^{48} Sc)} \; < \; 4.3 \times 10^{-12}. \tag{2}$$

In the present paper we study the light Majorana neutrino mechanism for the $(\mu^-, \mathbf{e}^+)$ conversion. Despite the previous rough estimates [2] indicate a very small branching ratio for this mode of $(\mu^-, \mathbf{e}^+)$ conversion, by far below the experimental bound (2), we undertake a detailed study of this mode for several reasons. First, the nuclear theory of $(\mu^-, \mathbf{e}^+)$ conversion is not yet sufficiently elaborated, as in the case of $0\nu\beta\beta$-decay, and requires further development. Second, $(\mu^-, \mathbf{e}^+)$ conversion may receive contribution from other mechanisms offered by various models beyond the SM such as the R-parity violating supersymmetric models, the leptoquark extensions of the SM etc. Some of these mechanisms may involve the light neutrino exchange and, therefore, from the view point of nuclear structure calculations they resemble the ordinary light neutrino mechanism. Thus our present study can be viewed as a first step towards a more general description of $(\mu^-, \mathbf{e}^+)$ conversion including all the possible mechanisms.

Below we develop a detailed nuclear structure theory for the light neutrino exchange mechanism of this process on the basis of the nuclear proton-neutron renormalized Quasi-particle Random Phase Approximation (pn-QRPA) wave functions [18, 19]. We perform a realistic calculation of the width of this process for the nuclear target ^{48}Ti using limits on

neutrino masses and mixings from neutrino oscillation phenomenology. A comparison with the previous estimations of $R_{\mu e^+}$ will be also presented [7, 11].

The paper is organized as follows. The possible values of Majorana neutrino masses and mixings are discussed in sect. 2. The amplitude and width of (μ^-, e^+) conversion are derived in sect. 3. The details of the calculation for the case of (μ^-, e^+) conversion in ^{48}Ti and our results are given in sect. 4. In sect. 5 we summarize our conclusions.

2 Majorana neutrino mass matrix

The finite masses of neutrinos are tightly related to the problem of lepton flavor/number violation. The Dirac, Majorona and Dirac-Majorana neutrino mass terms in the Lagrangian offer different neutrino mixing schemes and allow various lepton number/flavor violating processes [20, 21, 22]. The favored neutrino mixing schemes has to accommodate present neutrino phenomenology in a natural way, in particular, to answer the question of the smallness of neutrino masses compared to the charged lepton ones. The most prominent guiding principle in this problem is the see-saw mechanism which can be realized in various models beyond the SM. A generic neutrino mass term is given by the formula

$$\mathcal{L}^{D+M} = - \sum_{l,l'=e,\mu,\tau} [\, \frac{1}{2}\, \overline{(\nu'_{l'L})^c}\, (M^M_L)_{l'l}\, \nu'_{lL} + \frac{1}{2}\, \overline{\nu'_{l'R}}\, (M^M_R)_{l'l}\, (\nu'_{lR})^c + \overline{\nu'_{l'R}}\, (M^D)_{l'l}\, \nu'_{lL}\,] + h.c. \quad (3)$$

The first two terms do not conserve the total lepton number L. Here, ν'_L and ν'_R are the weak doublet and singlet flavor eigenstates. The indices L and R refer to the left-handed and right-handed chirality states, respectively, and the superscript c refers to the operation of charge conjugation. M^M_L and M^M_R are complex non-diagonal symmetrical 3x3 matrices. The flavor neutrino fields are superpositions of six Majorana fields ν_i with definite masses m_i. Yanagida, Gell-Mann, Ramond and Slansky suggested that the elements of M^D and M^M_L be comparable with the masses of charged leptons and the hypothetical scale of lepton number violation ($M_{LNV} \approx 10^{12} GeV$), respectively. Then by diagonalization of the Dirac-Majorana mass term one ends up with the three very light and three very heavy neutrino eigenstates. This is the celebrated see-saw mechanism. Enlarging the number of the right handed neutrino states ν_R one can introduce sterile light mass eigenstates which may play a certain role in the explanation of the neutrino oscillation data including the LSND results. However the active-sterile neutrino oscillations as a dominant channel seems to be disfavored according to a recent Super Kamiokande global analysis [23] and work in Tübingen [34].

Sticking to the three neutrino scenario one may try to reconstruct the corresponding mass matrix from the neutrino oscillation data. This requires certain assumptions on its structure or additional experimental data. Solar, atmospheric and LSND neutrino data give information on the neutrino mass square differences Δm^2_{ij} as well as on the mixing angles of the unitary matrix U [24, 25, 26, 27, 34] relating the weak ν'_{lL} and mass ν_{iL} neutrino eigenstates

$$\nu'_{lL} = \sum_{i=1}^{3} \; U^{(\nu)}_{li} \;\; \nu_{iL} \quad (l = \; e, \; \mu, \; \tau). \quad (4)$$

This information can be used to restore the neutrino mass matrix inverting its diagonalization as

$$\mathcal{M}^{ph} = U^{(\nu)} \cdot diag(m_1, m_2, m_3) \cdot U^{(\nu)T}, \quad (5)$$

if additional assumptions about the overall mass scale as well as about the CP phases $\zeta_{CP}^{(i)}$ of the neutrino mass eigenstates are made. This matrix can be identified with the 3×3 Majorana mass terms M_L^M in (3) and used in various phenomenological applications, for instance, in analysis of lepton number violating processes. The recent literature contains many sophisticated studies made in this direction (see, for instance, [28] and references therein).

The elements of the Majorana mass matrix are related to the effective Majorana neutrino masses $< m_\nu >_{\alpha\beta}$ $(\alpha, \beta = e, \mu, \tau)$ as

$$\mathcal{M}^{th}_{\alpha\beta} \equiv < m_\nu >_{\alpha\beta} = \sum_k^3 U^{(\nu)}_{\alpha k} \, U^{(\nu)}_{\beta k} \, \zeta^{(k)}_{CP} \, m_k, \tag{6}$$

neglecting mixing with heavy neutral states if they exist in the neutrino mass spectrum. Amplitudes of lepton number violating processes are proportional to the corresponding effective neutrino masses [2, 20]. Thus $0\nu\beta\beta$-decay amplitude is proportional to $< m_\nu >_{ee}$, the so called effective electron neutrino mass [7, 8]. From the currently most stringent lower limit on $0\nu\beta\beta$-decay half-life of ^{76}Ge $T^{0\nu}_{1/2} \geq 1.1 \times 10^{25} years$ [4] one obtains $< m_\nu >_{ee} < 0.62\ eV$ [29, 30]. The effective Majorana muon neutrino mass $< m_\nu >_{\mu\mu}$ is related with the light neutrino exchange modes of muonic analog of $0\nu\beta\beta$-decay [3], semipletonic decay of kaon $K^+ \to \pi^-\mu^+\mu^+$ [31, 32] etc. $< m_\nu >_{\mu e}$ enters the amplitude of the (μ^-, e^+) conversion [10, 11] and of the kaon decay into a muon and a positron $(K^+ \to \pi^-\mu^+e^+)$. Some other elements of $\mathcal{M}^{th}$ are associated with rare B-decays [33].

In Ref. [34] the maximal allowed values for the elements of $\mathcal{M}^{ph}$ have been deduced from solar, atmospheric, LSND data and restriction coming from $0\nu\beta\beta$-decay. The result is

$$\begin{pmatrix} < m_\nu >_{ee} & < m_\nu >_{e\mu} & < m_\nu >_{e\tau} \\ < m_\nu >_{\mu e} & < m_\nu >_{\mu\mu} & < m_\nu >_{\mu\tau} \\ < m_\nu >_{\tau e} & < m_\nu >_{\tau\mu} & < m_\nu >_{\tau\tau} \end{pmatrix} \leq \begin{pmatrix} 0.60 & 0.97 & 0.85 \\ 0.97 & 0.76 & 0.80 \\ 0.85 & 0.80 & 1.17 \end{pmatrix} \mathrm{eV}. \tag{7}$$

Thus in our analysis of the light neutrino exchange mode of the (μ^-, e^+) conversion process we shall assume $| < m_\nu >_{\mu e} | \leq 0.97\ eV$.

We shall also use a more conservative model independent estimate of the effective neutrino mass. Atmospheric and solar neutrino oscillation data show up $\Delta m^2 << (1eV)^2$ suggesting that all the neutrino mass eigenstates are approximately degenerate at the 1 eV scale [35]. This observation in combination with the tritium beta decay endpoint allows one to set upper bounds on masses of all the three neutrinos [35] $m_{e,\mu,\tau} \leq 3$eV. Thus in the three neutrino scenario one derives $|\langle m_\nu \rangle_{ij}| \leq 9$eV for $i, j = e, \mu, \tau$ [2, 32].

3 The $(\mu^-, \mathbf{e}^+)$ conversion mediated by light neutrinos

The process of $(\mu^-, \mathbf{e}^+)$ conversion is very similar to the $0\nu\beta\beta$-decay. Both processes violate lepton number by two units and take place only if the neutrino is a Majorana particle with non-zero mass. However, there are other important differences: i) The available energies for these two processes differ considerably. In addition, the number of leptons in final states is different. These facts result in significantly different phase space integrals. ii) The emitted positron in $(\mu^-, \mathbf{e}^+)$ conversion has large momentum and therefore the long-wave

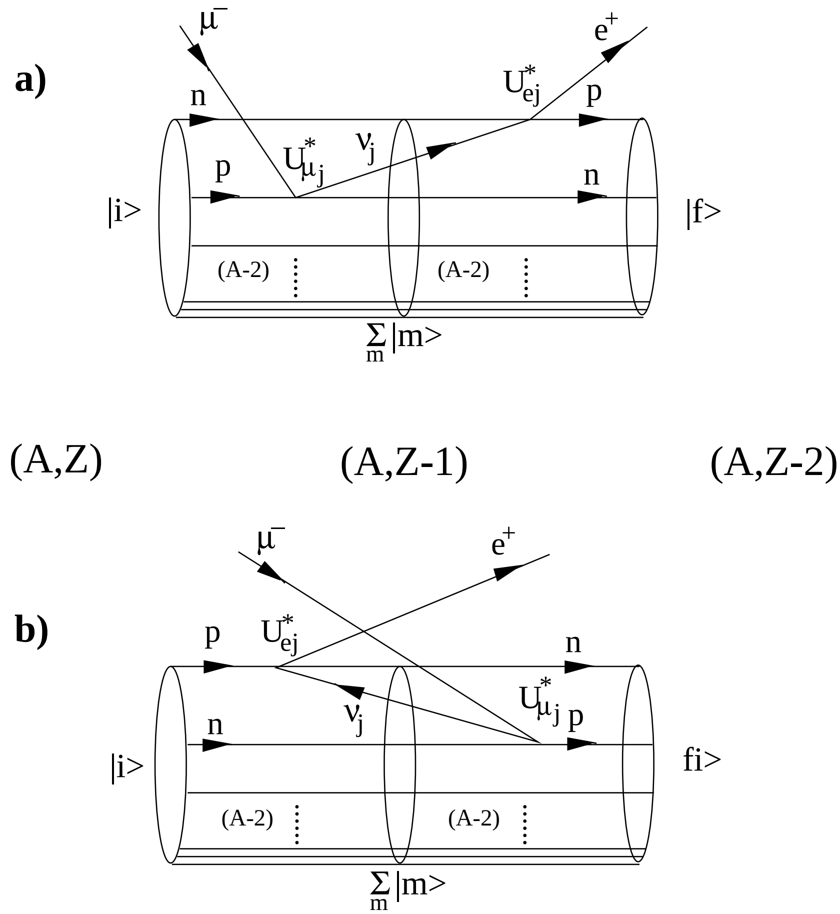

Figure 1: The direct (a) and cross (b) Feynmann diagrams of the (μ^-, e^+) conversion in nuclei mediated by Majorana neutrinos.

approximation is not valid. iii) As it will be shown below, in the case of light neutrino-exchange there is a singular behavior of the $(\mu^-, \mathbf{e}^+)$ nuclear matrix element which is the additional source of the difficulties for the numerical integration. iv) In the case of the $(\mu^-, \mathbf{e}^+)$ conversion there is a great number of nuclear final states. Nevertheless, the major contribution is here assumed to come from the transition to the ground state of the final nucleus.

We shall discuss the amplitude and width of (μ^-, e^+) conversion in nuclei mediated by light Majorana neutrinos. This process is shown in Fig. 1. We concentrate only on the nuclear transition connecting the ground states of the initial and final nuclei, which is favored from the experimental point of view due to the minimal background. In this case the e^+ spectrum has a peak at the energy

$$E_{e^+} = m_\mu - \varepsilon_b - (E_f - E_i). \tag{8}$$

Here, m_μ, ε_b, E_i and E_f are the mass of muon, the muon atomic binding energy (for ^{48}Ti $\varepsilon_b = 1.45\ MeV$), the energies of initial and final ground states, respectively. Latter on we that the kinetic energy of the final nucleus is negligible.

The weak interaction Hamiltonian in the neutrino mass eigenstate basis has the standard

form

$$\mathcal{H}^{weak}(x) = U_{li}^{(\nu)} \frac{G_F}{\sqrt{2}} \left[\bar{l}_L(x)\gamma_\alpha(1+\gamma_5)\nu_{iL}(x)\right] j_\alpha(x) + h.c. \quad (l \ = \ e, \ \mu, \ \tau), \tag{9}$$

where $j_\alpha(x)$ is the charged hadron current. The neutrino mixing matrix $U_{li}^{(\nu)}$ is defined in Eq. (4).

In second order of the weak-interaction we get for the (μ^-, e^+) conversion the following matrix element

$$\begin{aligned} < f|S^{(2)}|i > \ &= \ i\left(\frac{G_F}{\sqrt{2}}\right)^2 \ < m_\nu >_{\mu e} \ \frac{1}{(2\pi)^{3/2}} \frac{1}{\sqrt{4E_{\mu^-}E_{e^+}}} \ u^T(k_{e^+})C^{-1}(1+\gamma_5)u(k_{\mu^-}) \times \\ & \frac{2m_e m_\mu}{4\pi m_\mu \mathcal{R}} g_A^2 \ \mathcal{M}^{\Phi}_{<m_\nu>_{\mu e}} \ 2\pi\delta(E_{\mu^-} + E_i - E_f - E_{e^+}), \end{aligned} \tag{10}$$

where

$$\mathcal{M}^{\Phi}_{<m_\nu>_{\mu e}} = \frac{M_F^\Phi}{g_A^2} - M_{GT}^\Phi, \tag{11}$$

with

$$\begin{aligned} M_F^\Phi \ &= \ \frac{4\pi\mathcal{R}}{(2\pi)^3} \int \frac{d\vec{q}}{2q} \times \\ & \sum_n \left(\frac{< 0_i^+ | \sum_l \tau_l^+ e^{-i\vec{k}_{e^+}\cdot\vec{r}_l} e^{-i\vec{q}\cdot\vec{r}_l} |n >< n| \sum_m \tau_m^+ e^{i\vec{q}\cdot\vec{r}_m} \Phi(r_m) |0_f^+ >}{q - E_{\mu^-} + E_n - E_i + i\varepsilon_n} + \right. \\ & \left. \frac{< 0_i^+ | \sum_m \tau_m^+ e^{i\vec{q}\cdot\vec{r}_m} \Phi(r_m) |n >< n| \sum_l \tau_l^+ e^{-i\vec{k}_{e^+}\cdot\vec{r}_l} e^{-i\vec{q}\cdot\vec{r}_l} |0_f^+ >}{q + E_{e^+} + E_n - E_i + i\varepsilon_n} \right), \end{aligned} \tag{12}$$

$$\begin{aligned} M_{GT}^\Phi \ &= \ \frac{4\pi\mathcal{R}}{(2\pi)^3} \int \frac{d\vec{q}}{2q} \times \\ & \sum_n \left(\frac{< 0_i^+ | \sum_l \tau_l^+ \vec{\sigma}_l e^{-i\vec{k}_{e^+}\cdot\vec{r}_l} e^{-i\vec{q}\cdot\vec{r}_l} |n > \cdot < n| \sum_m \tau_m^+ \vec{\sigma}_m e^{i\vec{q}\cdot\vec{r}_m} \Phi(r_m) |0_f^+ >}{q - E_{\mu^-} + E_n - E_i + i\varepsilon_n} + \right. \\ & \left. \frac{< 0_i^+ | \sum_m \tau_m^+ \vec{\sigma}_m e^{i\vec{q}\cdot\vec{r}_m} \Phi(r_m) |n > \cdot < n| \sum_l \tau_l^+ \vec{\sigma}_l e^{-i\vec{k}_{e^+}\cdot\vec{r}_l} e^{-i\vec{q}\cdot\vec{r}_l} |0_f^+ >}{q + E_{e^+} + E_n - E_i + i\varepsilon_n} \right). \end{aligned} \tag{13}$$

Here, $\mathcal{R} = r_0 A^{1/3}$ is the mean nuclear radius, with $r_0 = 1.1 fm$ and m_e is the mass of electron. $\vec{r}_i$ is a coordinate of the ith nucleon. E_{μ^-} (k_{μ^-}) and E_{e^+} (k_{e^+}) denote energies (four-momenta) of the bound muon ($E_{\mu^-} = m_\mu - \varepsilon_b$) and the emitted positron, respectively. E_n and ε_n are respectively energy and width of the intermediate nuclear state. $\Phi(r)$ is the radial part of bound muon in its orbit (see Appendix A).

In the derivation of nuclear matrix element $\mathcal{M}^{\Phi}_{<m_\nu>_{\mu e}}$ we neglected the contribution from higher order terms of nucleon current (weak-magnetism, induced pseudoscalar coupling), which are expected to play a less important role. Following the analysis in Ref. [29] their consideration can reduce the value of $\mathcal{M}^{\Phi}_{<m_\nu>_{\mu e}}$ by an amount of about 20% by analogy to the $0\nu\beta\beta$-decay.

We have normalized the nuclear matrix element $\mathcal{M}^{\Phi}_{<m_\nu>_{\mu e}}$ in the same way as usual for the corresponding $0\nu\beta\beta$-decay matrix element. We note that the denominators in the expressions for the (μ^-, e^+) conversion and the $0\nu\beta\beta$-decay exhibit a different behavior. This is because the energy of the bound muon E_{μ^-} is large. The two denominators in Eq. (13) can be associated with the direct and the cross Feynman diagrams in Fig. 1. One notes that the value of $(-E_{\mu^-} + E_n - E_i)$ is negative. This fact implies that the widths of the nuclear states play an important role and that the imaginary part of the nuclear matrix element can be large. This point was not discussed in previous publications [7, 9, 10, 11] and is one of the motivations of our (μ^-, e^+) conversion calculation. We want to investigate if this singular behavior of the amplitude can lead to an enhancement of the (μ^-, e^+) conversion branching ratio or not. In order to simplify the numerical calculations we complete the sum over virtual intermediate nuclear states by closure after replacing E_n, ε_n by some average values $< E_n >$, ε, respectively:

$$\begin{aligned} \sum_n \frac{|n><n|}{q - E_{\mu^-} + E_n - E_i + i\varepsilon_n} &= \frac{1}{q - E_{\mu^-} + < E_n > - E_i + i\varepsilon}, \\ \sum_n \frac{|n><n|}{q + E_{e^+} + E_n - E_i + i\varepsilon_n} &= \frac{1}{q + E_{e^+} + < E_n > - E_i + i\varepsilon} \end{aligned} \qquad (14)$$

Next we assume that the muon wave function varies very little inside the nuclear system, i.e., the following approximation is used

$$|\mathcal{M}^{\Phi}_{<m_\nu>_{\mu e}}|^2 \ = < \Phi_\mu >^2 \ |\mathcal{M}_{<m_\nu>_{\mu e}}|^2. \qquad (15)$$

The explicit form of $< \Phi_\mu >^2$ is given in Appendix B.

For the width of (μ^-, e^+) conversion we obtain

$$\Gamma_{<m_\nu>_{\mu e}} = \frac{1}{\pi} \ E_{e^+} \ k_{e^+} \ F(Z-2, E_{e^+}) \ c_{\mu e} \ < \Phi_\mu >^2 \ |\mathcal{M}_{<m_\nu>_{\mu e}}|^2 \ \left| \frac{< m_\nu >_{\mu \ e}}{m_e} \right|^2, \qquad (16)$$

where $c_{\mu e} = 2G_F^4[(m_e m_\mu)/(4\pi m_\mu \mathcal{R})]^2 g_A^4$ and $k_{e^+} = |\vec{k}_{e^+}|$. The nuclear matrix element $\mathcal{M}_{<m_\nu>_{\mu e}}$ can be decomposed into the contributions coming from direct and crossed Feynmann diagrams in Fig. 1 as

$$\mathcal{M}_{<m_\nu>_{\mu e}} = M^{dir.} + M^{cro.}, \qquad (17)$$

where

$$\begin{aligned} M^{dir.} &= < 0_i^+ | \sum_{kl} \tau_k^+ \tau_l^+ 4\pi \sum_\lambda (-1)^\lambda \sqrt{2\lambda+1} \ j_\lambda(k_{e^+} R_{kl}) \ \{Y_\lambda(\Omega_r) \otimes Y_\lambda(\Omega_R)\}_0 \times \\ & \frac{\mathcal{R}}{\pi} \int_0^\infty \frac{j_0(q r_{kl}) j_\lambda(k_{e^+} r_{kl}/2)}{q - E_{\mu^-} + E_n - E_i + i\varepsilon} (\vec{\sigma_k} \cdot \vec{\sigma_l} f_A^2(q^2) - \frac{f_V^2(q^2)}{g_A^2}) q dq \ |0_f^+ > \\ M^{cro.} &= < 0_i^+ | \sum_{kl} \tau_k^+ \tau_l^+ 4\pi \sum_\lambda (-1)^\lambda \sqrt{2\lambda+1} \ j_\lambda(k_{e^+} R_{kl}) \ \{Y_\lambda(\Omega_r) \otimes Y_\lambda(\Omega_R)\}_0 \times \\ & \frac{\mathcal{R}}{\pi} \int_0^\infty \frac{j_0(q r_{kl}) j_\lambda(k_{e^+} r_{kl}/2)}{q + E_{e^+} + E_n - E_i + i\varepsilon} (\vec{\sigma_k} \cdot \vec{\sigma_l} f_A^2(q^2) - \frac{f_V^2(q^2)}{g_A^2}) q dq \ |0_f^+ >, \end{aligned} \qquad (18)$$

Table 1: Nuclear matrix elements of the light Majorana neutrino exchange mode of the (μ^-, e^+) conversion in ^{48}Ti [see Eqs. (17) and (18)]. The calculations have been performed within pn-RQRPA without and with consideration of two-nucleon short-range correlations (s.r.c.).

	without s.r.c				with s.r.c			
g_{pp}	$M^{cro.}$ $[10^{-2}]$	$R(M^{dir.})$ $[10^{-2}]$	$I(M^{dir.})$ $[10^{-2}]$	$\lvert\mathcal{M}_{<m_\nu>_{\mu e}}\rvert$ $[10^{-2}]$	$M^{cro.}$ $[10^{-2}]$	$R(M^{dir.})$ $[10^{-2}]$	$I(M^{dir.})$ $[10^{-2}]$	$\lvert\mathcal{M}_{<m_\nu>_{\mu e}}\rvert$ $[10^{-2}]$
0.80	9.65	0.23	8.83	13.2	4.88	-7.99	4.98	5.87
1.00	7.71	3.36	5.88	12.5	3.40	-4.03	2.37	2.45
1.20	5.05	9.09	1.78	14.2	1.30	2.71	-1.32	4.22

with

$$\vec{r}_{ij} = \vec{r}_i - \vec{r}_j, \quad r_{ij} = |\vec{r}_{ij}|, \quad \vec{R}_{ij} = \vec{r}_i + \vec{r}_j, \quad R_{ij} = |\vec{R}_{ij}|. \tag{19}$$

For the normalized nucleon form factors we use the conventional dipole form $f_V(q^2) = 1/(1+q^2/\Lambda_V^2)^2$ $[\Lambda_V^2 = 0.71\ (GeV)^2]$, $f_A(q^2) = 1/(1+q^2/\Lambda_A^2)^2$ $[\Lambda_A = 1.09\ GeV]$.

4 Results and discussions

The nuclear matrix elements of the (μ^-, e^+) conversion process have been calculated within the proton-neutron renormalized Quasiparticle Random Phase Approximation [18, 19, 36, 37]. The considered single-particle model space both for protons and neutrons have been as follows: The full $0-3\hbar\omega$ shells plus $2s_{1/2}$, $0g_{7/2}$ and $0g_{9/2}$ levels. The single particle energies were obtained by using a Coulomb–corrected Woods–Saxon potential. Two-body G-matrix elements were calculated from the Bonn one-boson exchange potential within the Brueckner theory. The pairing interactions have been adjusted to fit the empirical pairing gaps [38]. The particle-particle and particle-hole channels of the G-matrix interaction of the nuclear Hamiltonian H are renormalized by introducing the parameters g_{pp} and g_{ph}, respectively. The calculations have been performed for $g_{ph} = 1.0$ and $g_{ph} = 0.8,\ 1.0,\ 1.2$. The two-nucleon correlation effect has been considered in the same way as in Ref. [29, 37].

In calculation of the (μ^-, e^+) conversion nuclear matrix elements we have used the fact that the widths of the low lying nuclear states are negligible in comparison with their energies. Therefore we have carried out the calculation in the limit $\varepsilon \to 0$ using the formula

$$\frac{1}{\alpha + i\varepsilon} = \mathcal{P}\frac{1}{\alpha} - i\pi\delta(\alpha), \tag{20}$$

which allows one to separate the real and imaginary parts of the (μ^-, e^+) conversion amplitude.

In Table 1 nuclear matrix elements of the light Majorana neutrino exchange mechanism of the (μ^-, e^+) conversion in ^{48}Ti are presented. The adopted value of $< E_n > -E_i$ was 10 MeV. We have found that our results depend weakly on this average value of the nuclear states within the interval $2\ MeV \ \leq (< E_n > -E_i) \leq \ 15\ MeV$. However, they depend significantly on the details of nuclear model, in particular, on the renormalization of the particle-particle channel of the nuclear Hamiltonian, and on the two-nucleon short-range

correlation effect (s.r.c.). A new feature of this (μ^-, e^+) conversion calculation is that the imaginary part of $\mathcal{M}_{<m_\nu>_{\mu e}}$ is significant, i.e., can not be neglected. This fact was not noticed in the previous (μ^-, e^+) calculations [7, 9, 10, 11].

In the further analysis we shall consider the nuclear matrix element $|\mathcal{M}_{<m_\nu>^{\mu e}}|$ obtained for $g_{pp} = 1.0$ by considering the two-nucleon short range correlations. It is interesting to compare its value with the value of $0\nu\beta\beta$-decay matrix elements for A=48 nuclear system. We have

$$|\mathcal{M}_{<m_\nu>_{\mu e}}| = 2.45 \times 10^{-2}, \qquad |\mathcal{M}_{<m_\nu>_{ee}}| = 0.82. \tag{21}$$

We see that the matrix element for the (μ^-, e^+) conversion is strongly suppressed in comparison with $0\nu\beta\beta$-decay matrix element by about factor of 400. It is mostly due to the large momentum of the outgoing positron in the (μ^-, e^+) conversion process.

One can compare also the width of (μ^-, e^+) conversion in ^{48}Ti with the width of $0\nu\beta\beta$-decay of ^{48}Ca. We get

$$\begin{aligned}
\frac{\Gamma_{<m_\nu>_{\mu e^+}}}{\Gamma_{<m_\nu>_{ee}}} &= \frac{ln(2)}{G_{01}}\frac{1}{\pi}\; E_{e^+}\; K_{e^+}\; F(Z-2, E_{e^+})\; c_{\mu e}\; <\Phi_\mu>^2 \;\frac{|\mathcal{M}_{<m_\nu>_{\mu e}}|^2}{|\mathcal{M}_{<m_\nu>_{ee}}|^2}\; \left|\frac{<m_\nu>_{\mu e}}{<m_\nu>_{ee}}\right|^2, \\
&= 1.97 \times 10^5\; \frac{|\mathcal{M}_{<m_\nu>_{\mu e}}|^2}{|\mathcal{M}_{<m_\nu>_{ee}}|^2}\; \left|\frac{<m_\nu>_{\mu e}}{<m_\nu>_{ee}}\right|^2, \\
&= 176.\; \left|\frac{<m_\nu>_{\mu e}}{<m_\nu>_{ee}}\right|^2.
\end{aligned} \tag{22}$$

The width of the (μ^-, e^+) conversion is enhanced mostly due to the larger available energy for this process. A comparison with the width of $0\nu\beta\beta$-decay show that it is disfavored by smaller coulombic factor F(Z,E) ($\sim 0.623/1.8$) and by significantly smaller value of associated nuclear matrix element ($\sim (0.0245/0.82)^2$). If we assume the effective neutrino masses $< m_\nu >_{\mu e}$ and $< m_\nu >_{ee}$ to be comparable [see Eq. (7)] we find that $\Gamma_{<m_\nu>_{\mu e^+}}$ is enhanced by a factor of about 200 in comparison with $\Gamma_{<m_\nu>_{ee}}$. We have used $G_{01} = 8.031 \times 10^{-14}\ year^{-1}$ [39].

From the experimental point of view it is interesting to compare the (μ^-, e^+) conversion width with the width of ordinary muon capture rate. We have

$$\begin{aligned}
\frac{\Gamma_{<m_\nu>_{\mu e}}}{\Gamma_\mu} &= 2\frac{E_{e^+}\; k_{e^+}}{m_\mu^2}\frac{c_{\mu\mu}}{G_F^2}\frac{|\mathcal{M}_{<m_\nu>_{\mu e}}|^2 F(Z-2, E_{e^+})}{[G_V^2 + 3G_A^2 + G_P^2 - 2G_A G_P] Z f(Z, A)}\left|\frac{<m_\nu>_{\mu e}}{m_e}\right|^2 \\
&= 2.24 \times 10^{-22} |\mathcal{M}_{<m_\nu>_{\mu e}}|^2 \left|\frac{<m_\nu>_{\mu e}}{m_e}\right|^2 \\
&= 1.34 \times 10^{-25} \left|\frac{<m_\nu>_{\mu e}}{m_e}\right|^2
\end{aligned} \tag{23}$$

If we use the prediction for $< m_\nu >_{\mu e}$ coming from neutrino oscillation phenomenology, i.e., $< m_\nu >_{\mu e}\ \leq\ 0.97\ eV$ and more conservative bound $< m_\nu >_{\mu e}\ \leq\ 9\ eV$ [see Eq. (7)], we end up with

$$\frac{\Gamma_{<m_\nu>_{\mu e}}}{\Gamma_\mu} = 4.8 \times 10^{-37},\ 4.5 \times 10^{-35}, \tag{24}$$

This value is about ten orders of magnitude smaller as the estimated one for the (μ^-, e^+) conversion in ^{32}S by Doi et al. [7]. There could be various reasons for this difference. First,

the $< m_\nu >_{\mu e}$ nuclear matrix element calculated by Doi et al. contains contributions from all final nuclear states and not only from the $0^+_{g.s.} \rightarrow 0^+_{g.s.}$ transition as in our case. It can be that the experimentally interesting $g.s. \rightarrow g.s.$ transition exhausts only a small part from all allowed nuclear transitions. Second, in the simplified calculation of Doi et al. nuclear matrix elements were evaluated by summing the final nuclear states with closure. It usually leads to overestimation of the results as we know from calculation of ordinary muon capture. Third, the nuclear matrix element of Ref. [7] have been evaluated by using the long–wave approximation. Our comparison of the (μ^-, e^+) conversion and the $0\nu\beta\beta$-decay (long–wave approximation is used) matrix elements shows that it can lead to overestimation of $\mathcal{M}_{<m_\nu>_{\mu e}}$ by factor up to 10^2. Fourth, the problem of the ground and short–range correlations have been not addressed in Ref. [7]. We have found that $\mathcal{M}_{<m_\nu>_{\mu e^+}}$ matrix element for A=48 nuclear system is strongly suppressed by both of them. It is not clear whether this effect is due to the chosen target. However, we note that $\mathcal{M}_{<m_\nu>_{\mu e^+}}$ for the conversion in ^{48}Ti consists of transition to the doubly closed shell nucleus ^{48}Ca, which, e.g., in the case of $0\nu\beta\beta$-decay are less favored. To clarify this issue, the calculations of the (μ^-, e^+) conversion for other nuclear systems are necessary.

It is worthwhile to notice that our result is in relative good agreement with the calculations performed by Leontaris and Vergados [11] for the (μ^-, e^+) conversion in ^{58}Ni. By using the same value for effective Majorana neutrino mass $< m_\nu >_{\mu e}$ as in this article the result of Ref. [11] corresponds to a branching ratio equal to 3.2×10^{-36} relative to the total absorption of the muon for the ground state to ground state transition. The difference of about one order with our result for A=48 can be attributed to the nuclear physics aspect of the (μ^-, e^+) conversion, i.e., to a given nuclear system and the chosen nuclear model. We also remark that in Ref. [11] the imaginary part of the (μ^-, e^+) conversion amplitude was not considered.

5 Summary and outlook

In summary, the lepton number violating process of (μ^-, e^+) conversion in nuclei have been studied. The light Majorana neutrino–exchange mechanism of this process have been considered. A detailed analysis of this mode of the (μ^-, e^+) conversion have been performed. The first realistic calculation of the $0^+_{g.s.} \rightarrow 0^+_{g.s.}$ channel of this process, which is most favored for experimentally studies, are presented. The relevant matrix elements for A=48 nuclear system have been calculated within the pn-RQRPA. The effects of the ground state and two-nucleon short-range correlations have been analyzed. It was found that by inclusion of them the value of (μ^-, e^+) conversion matrix elements is strongly suppressed. We are the first, to our knowledge, showing that the imaginary part of the nuclear matrix element is large and should be taken into account. Further, a comparison of different relevant aspects with the $0\nu\beta\beta$-decay process are presented. It is shown that the width of (μ^-, e^+) conversion is about by factor of 200 larger as that of $0\nu\beta\beta$-decay by assuming predictions for effective neutrino masses coming from neutrino oscillation phenomenology. Nevertheless, the studied neutrino exchange mode of lepton number violating (μ^-, e^+) conversion is not suitable for experimental study being extremely small compared to the ordinary muon capture. This fact, however, does not disfavor further experimental study of the (μ^-, e^+) conversion in nuclei as some other lepton number violating mechanisms, e.g., those coming from GUT's

and SUSY models, can dominate this process. Therefore, they should be carefully examine too.

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6 Appendix A

The bound muon wave function (1S-state) is

$$\Psi_\mu(x) = \Phi_\mu(\vec{x}) e^{-iE_{\mu^-} x_0} \frac{u_\mu^s}{\sqrt{2E_{\mu^-}}}, \tag{25}$$

where

$$\begin{aligned} \Phi_\mu(\vec{x}) &= \frac{Z^{3/2}}{(\pi a_\mu^3)^{1/2}} e^{-Z|\vec{x}|/a_\mu}, \\ u_\mu^s &= \begin{pmatrix} \chi^s \\ 0 \end{pmatrix} \sqrt{2E_{\mu^-}} \end{aligned} \tag{26}$$

with $a_\mu = 4\pi/(m_\mu e^2)$ $(a_\mu/a_e \approx m_e/m_\mu \approx 5 \times 10^{-3})$ m_μ is the reduced mass of muon nucleus system.

7 Appendix B

The width for the ordinary muon capture reaction $\mu^- + (Z, A) \to \nu_\mu + (Z-1, A)$ can be written in the Primakoff form [40]

$$\Gamma_\mu = \frac{1}{2\pi} m_\mu^2 (G_F \cos\theta_c)^2 < \Phi_\mu >^2 Z[G_V^2 + 3G_A^2 + G_P^2 - 2G_A G_P] f(Z, A), \tag{27}$$

where muon average probability density over the nucleus is

$$< \Phi_\mu >^2 \equiv \frac{\int |\Phi_\mu(\vec{x})|^2 \rho(\vec{x}) d^3x}{\int \rho(\vec{x}) d^3x}. \tag{28}$$

$\rho(\vec{x})$ is the nuclear density. To a good approximation it has been found

$$< \Phi_\mu >^2 = \frac{\alpha^3 m_\mu^3}{\pi} \frac{Z_{eff}^4}{Z}, \tag{29}$$

i.e., the deviation from the behavior of the wave function at the origin has been taken into account by the effective proton number Z_{eff}. The values of this effective charge has been

calculated for the nuclear systems of interest in Ref. [12]. In particular, one finds $Z_{eff} = 17.6$ for $Z = 22$. The quadratic combination of the weak coupling constants is

$$[G_V^2 + 3G_A^2 + G_P^2 - 2G_A G_P] \approx 5.9. \tag{30}$$

The function $f(Z, A)$ takes into account the two-nucleon correlations given by [11]

$$f(A, Z) = 1 - 0.03\frac{A}{2Z} + 0.25(\frac{A}{2Z} - 1) + 3.24(\frac{Z}{2A} - \frac{1}{2} - |\frac{1}{8Z} - \frac{1}{4A}|). \tag{31}$$

This Pauli blocking factor for ^{48}Ti takes the value $f(22, 48) = 0.11$.

References

[1] J. Schechter and J.W.F. Valle, Phys.Rev. D 25 (1982) 2951.

[2] C. Dib, V. Gribanov, S. Kovalenko and I. Schmidt, to appear in the Proceedings of NANPino conference, Dubna, Russia, July 19-22, 2000; hep-ph/hep-ph/0011213.

[3] J.H. Missimer, R.N. Mohapatra and N.C. Mukhopadhyay, Phys. Rev. D 50 (1994) 2067.

[4] L. Baudis et al., Phys.Rev.Lett. **83** (1999) 411.

[5] H. Ejiri, Phys. Rep. 338 (2000) 265.

[6] H.V. Klapdor-Kleingrothaus, *Springer Tracts in Modern Physics* **163** (2000) 69.

[7] M. Doi, T. Kotani, E. Takasugi, Prog. Theor. Phys. Suppl. 83 (1985) 1.

[8] A. Faessler and F. Šimkovic, J. Phys. G 24 (1998) 2139.

[9] A.N. Kamal and J.N. Ng, Phys. Rev. D 20 (1979) 2269.

[10] J.D. Vergados, Phys. Rev. D 23 (1981) 703, Phys. Rep. 133 (1986) 1.

[11] G.K. Leontaris and J.D. Vergados, Nucl. Phys. B 224 (1983) 137.

[12] T.S. Kosmas, G.K. Leontaris and J.D. Vergados, Prog. Part. Nucl. Phys. 33 (1994) 397.

[13] T.S. Kosmas, Amand Faessler and J.D. Vergados, J. Phys. G 23 (1997) 693.

[14] A. Faessler, T.S. Kosmas, S.G. Kovalenko, J.D. Vergados, Nucl. Phys. B 587 (2000) 25.

[15] T. Siiskonen, J. Suhonen, T.S. Kosmas, Phys. Rev. C 62 (2000) 035502.

[16] C. Dohmen et al, SINDRUM II Collaboration, Phys. Lett. B 317 (1993) 631.

[17] W. Honecker et al, SINDRUM II Collaboration, Phys. Rev. Lett. 76 (1996) 200.

[18] J. Toivanen and J. Suhonen, Phys. Rev. Lett. 75 (1995) 410.

[19] J. Schwieger, F. Šimkovic, and A. Faessler, Nucl. Phys. A 600 (1996) 179.

[20] S.M. Bilenky and S.T. Petcov, Rev. Mod. Phys. 59 (1987) 671.

[21] S.M. Bilenky, C. Giunti, W. Grimus, Prog. Part. Nucl. Phys. 43 (1999) 1.

[22] K. Zuber, Phys. Rep. 305 (1998) 295.

[23] Super-Kamiokande Collab., S. Fukuda *et al.*, Phys. Rev. Lett. 85 (2000) 3999.

[24] S.M. Bilenky, C. Guinty, W. Grimus, B. Kayser, and S.T. Petcov, Phys. Lett. B 465 (1999) 193.

[25] F. Vissani, JHEP 9906 (1999) 22.

[26] V. Barger and K. Whisnant, Phys. Lett. B 456 (1999) 194.

[27] A.Yu. Smirnov, Nucl. Phys. Proc. Suppl. 87 (2000) 288.

[28] W. Rodejohann, hep-ph/0003149; hep-ph/0008044; H.V. Klapdor-Kleingrothaus, H. Päs and A.Yu. Smirnov, hep-ph/0003219 and references therein.

[29] F. Šimkovic, G. Pantis, J.D. Vergados, and A. Faessler, Phys. Rev. C 60 (1999) 055502.

[30] A. Faessler, F. Šimkovic, Phys. At. Nucl. 63 (2000) 1165, hep-ph/9909252.

[31] L.S. Littenberg and R.E. Shrock, hep-ph/0005285.

[32] C. Dib, V. Gribanov, S. Kovalenko, I. Schmidt, Phys. Lett. B 493 (2000) 82.

[33] K. Zuber, Phys. Lett. B 479 (2000) 33.

[34] O. Haug, J.D. Vergados, Amand Faessler, S. Kovalenko, Nucl. Phys. B 565 (2000) 38.

[35] F. Vissani, hep-ph/9708483; V. Barger, T.J. Weiler, and K. Whisnant, Phys. Lett. B 442 (1998) 255.

[36] A. Faessler, S. Kovalenko, F. Šimkovic, and J. Schwieger, Phys. Rev. Lett. 78 (1997) 183 (1997); A. Faessler, S. Kovalenko, and F. Šimkovic, Phys. Rev. D 58 (1998) 115004.

[37] A. Wodecki, W.A. Kamiński, and F. Šimkovic, Phys. Rev. D 60 (1999) 11507.

[38] M.K. Cheoun, Amand Faessler, F. Šimkovic, G. Teneva, A. Bobyk, Nucl.Phys. A 561 (1993) 74.

[39] G. Pantis, F. Simkovic, J.D. Vergados, A. Faessler, Phys. Rev. C 53 (1996) 695.

[40] H. Primakoff, Rev. Mod. Phys. 31 (1959) 802; B. Goulard and H. Primakoff, Phys. Rev. C 11 (1975) 1894.